

# Robust Control of Thermal Mixing Process using Sliding Mode Control

Tabassum Rasul<sup>1</sup> and Monisha Pathak<sup>2</sup>

<sup>1</sup>Department of Electrical Engineering Jorhat Engineering College Jorhat, Assam, India, 785007

<sup>2</sup>Department of Instrumentation Engineering Jorhat Engineering College Jorhat, Assam, India, 785007

E-mail: <sup>1</sup>tabassum0205@gmail.com, <sup>2</sup>monisha.pathak@jecassam.ac.in

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**Abstract**—The idea of robust control of a thermal mixer by utilising sliding mode control technique is put forward in this paper. It is intended to design a Sliding Mode Controller to achieve the reference temperature of the mixer in presence of unsettling disturbances. The performance of the procedure is outlined by simulation in MATLAB environment. The designed controller ensures the convergence and stability requirement in presence of disturbances and hence proves its robustness. The outcome of the suggested procedure in comparison with classical PID control method shows that the proposed procedure is simple and successful over an extensive span of working conditions, model instabilities and nonlinear dynamics of the system.

## 1. INTRODUCTION

Mixers are generally utilised as a part of pharmaceuticals, chemicals, food processing industries for delivering different substrates with high included quality. Due to the inherent nonlinear qualities, presence of plant uncertainties and absence of consistent working conditions, control of chemical processes like mixing tank has always been a fascinating subject to control engineers. The issue of mixing the measures of hot and cool liquid to deliver a blend having a required temperature is well known to everybody, including individuals who are not control specialists. This is an issue which shows up in industrial systems as well as in everyday shower case [9,10]. In this paper we consider the first order plus dead time model (FOPDT) of the mixing tank and apply a robust control method to attain a desired tank temperature. Sliding Mode Control (SMC) is capable of controlling nonlinear systems because of its robustness to special kind of uncertainties and parameter variations [1,15]. Uses of SMC to various chemical processes have been accounted in [1,8,17].

The paper is arranged in the following manner. Section II describes the process considered. Section III portrays the underlying idea of the two control procedures applied. Section IV demonstrates the designing of the Sliding mode controller along with the tuning equations using system model based on FOPDT. Section V describes the simulation of the designed controller and presents the result obtained finally section VI presents the conclusion derived.

## 2. PROCESS

Mixing process is a vital procedure for quality generation of products in process industries. Various reactants are mixed in different concentrations to form products of different qualities which are used for further procedures. In some cases, a small blunder in the synthesis procedure could result in a poor quality product. The mixer consists of a mixing tank, a stirrer and valves wherein a hot water and a cold water flux are mixed together. The objective of the process is to keep up the temperature of the tank near its set point[1,9,10].

## 3. CONTROLLERS

### 3.1 PID Control

P, PI or PID are the customarily used controllers in process industries [3,6,16]. The PID control is described by:

where  $e$  is the deviation of the output variable,  $x(t)$  from the reference variable,  $r(t)$ . That is error

$$e(t) = r(t) - x(t) \quad (2)$$

The control signal  $u(t)$  is a total of three terms, the term relative to the error  $e(t)$  is the proportional term, the term relative to the integral of the error is the integral term and the derivative term corresponding to the derivative of the error. Stability can be ensured using only the proportional term. But proportional action results in offset error in the output. The integral term can eliminate the offset but increases overshoot and settling time. This can be decreased with increase in the derivative time ( $T_d$ ). The derivative term is used to provide damping to the response.

### 3.2 Sliding Mode Control (SMC)

Sliding mode control (SMC) is a robust nonlinear control method [1,2,3,4,5,15]. It is suitable for nonlinear dynamical systems working under conditions of uncertainties like parameter variations, modeling uncertainties and external disturbances.

Sliding mode control technique is computationally simple and is uncaring to matched uncertainties and outside aggravations. It is widely used for reduced order modeling of plant dynamics and has a finite-time convergence.

Sliding Mode Control has evolved from Variable Structure Control (VSC) [4,5,12].

The concept of SMC technique is to characterize a surface called sliding surface along which the system trajectory slides to its equilibrium point. A discrete control law is derived which drives the system trajectory on to the sliding surface and then slide along the surface to the final equilibrium point. The design procedure includes two steps. Firstly designing the sliding surface according to our requirement to ensure system stability and secondly formulating the control law in accordance to which the system's trajectory would converge to the sliding surface in finite time.

Fig. 1. shows the sliding mode control rule [1,3].

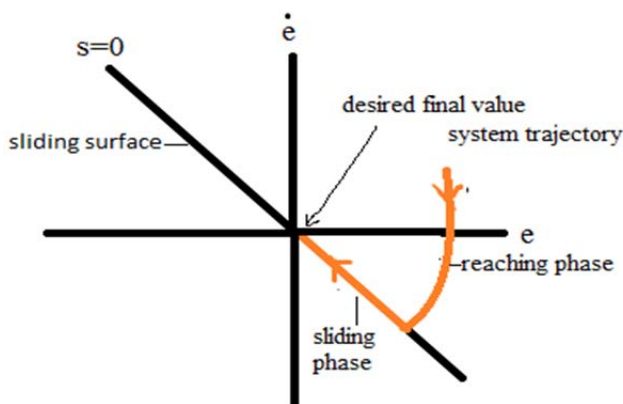


Fig. 1: The Concept of Sliding Mode Control

To the extent chemical processes are concerned firstly, constructing a complete model is troublesome because of the unpredictability of the process and various unknown parameters affecting the process. And secondly, mathematical equations describing the process dynamics are generally of higher order. The paper presents a viable approach utilizing a reduced order model of the chemical process.

The equation of the sliding surface is derived using an FOPDT model of the process [1,3,4,5,15].

The sliding surface equation is an integro-differential equation of the error and is given by

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t) dt \quad (3)$$

where  $e(t)$  as mentioned in (2) is the tracking error.

The tuning parameter  $\lambda$  decides the execution of the system on the sliding surface,  $n$  is the order of the system. The control objective is to ensure that the output is equal to the set point at

all instants of time, that is the error and its differentiation with time are zero. When the reference point is attained,  $S(t)$  becomes a constant value. To maintain a consistent value of  $S(t)$ ,  $e(t)$  must be zero at all times which means

$$\frac{d}{dt} S(t) = 0 \quad (4)$$

Once the sliding surface is designed, the control law is derived which directs the system trajectory to its final equilibrium point and satisfies (4). The SM Control law,  $u(t)$ , consists of a continuous part,  $u_c(t)$ , and a discrete part,  $u_d(t)$ . That is

$$u(t) = u_c(t) + u_d(t) \quad (5)$$

The continuous part is a function of the controlled variable,  $x(t)$ , and the reference variable,  $r(t)$  and is given by

$$u_c(t) = f(x(t), r(t)) \quad (6)$$

The discontinuous part,  $u_d(t)$  involves a switching function and switches about the sliding surface.

$$u_d(t) = K_d \frac{S(t)}{|S(t)| + \delta} \quad (7)$$

where  $K_d$  is the controller gain responsible for the reaching phase.  $\delta$  is the parameter responsible to eliminate chattering. Chattering is an undesirable high frequency oscillation in sliding motion which degrades the control objective. Perfect tracking can be achieved by reducing chattering. Thus the control law directs the system trajectory to the sliding surface, and then proceeds until the equilibrium state is reached.

## 4. DESIGNING THE CONTROLLER

### 4.1 Designing of SMCr from an FOPDT model of the process:

In this section, a SMCr structure for self regulating processes build on the FOPDT model of the actual process is outlined [1,3,15].

The first order plus dead time approximation of the system is given by

$$\frac{X(s)}{U(s)} = \frac{K_1 e^{-t_0 s}}{\tau s + 1} \quad (8)$$

where Laplace transform of the controlled variable  $X(s)$  and Laplace transform of the controller output  $U(s)$  are deviation variables. The model parameters that are the process gain,  $K_1$ , process time constant,  $\tau$  and process dead time,  $t_0$ , are used to obtain the primary evaluations of the tuning parameters of the SMCr.

The controller is developed using a reduced order model of the actual process. Hence the dead time term of the process model equation is evaluated using a first order Taylor series approximation which results in [10,14]

$$e^{-t_0 s} = \frac{1}{t_0 s + 1} \quad (9)$$

**4.2 Development of SMCr from a first order Taylor series approximation**

Applying Taylor series expansion and substituting (9) in (8) gives

$$\frac{X(s)}{U(s)} = \frac{K_1}{(\tau s + 1)(t_0 s + 1)} \tag{10}$$

In differential equation form,

$$t_0 \tau \ddot{x} + (t_0 + \tau) \dot{x} + x = ku \tag{11}$$

and since this is a second order differential equation, n=2, from (3) S becomes

$$S = \dot{e} + 2\lambda e + \lambda^2 \int_0^t e dt \tag{12}$$

From Eq. (4)

$$\dot{S} = \ddot{e} + 2\lambda \dot{e} + \lambda^2 e = 0 \tag{13}$$

Substituting e=r-x, in the first term of equation (13) yields

$$(\ddot{r} - \ddot{x}) + 2\lambda \dot{e} + \lambda^2 e = 0 \tag{14}$$

Solving (9) and substituting it in (12) gives u<sub>c</sub>, the continuous part of the control law as

$$u_c = \left(\frac{t_0 + \tau}{t_0 \tau}\right) \left[ \left(\frac{t_0 + \tau}{t_0 \tau} \dot{x} + \frac{x}{t_0 \tau} + \lambda^2 e + \ddot{r} + \lambda^2 \dot{r}\right) \right] \tag{15}$$

The procedure of deriving the expression for the continuous part of the control law from (12) and (14) is known as the equivalent control procedure in SMC theory [2].

The derivative of the reference value gives zero. Thus,

$$u_c = \left(\frac{t_0 + \tau}{K_1}\right) \left[ \left(\frac{t_0 + \tau}{t_0 \tau} - 2\lambda\right) \dot{x} + \frac{x}{t_0 \tau} + \lambda^2 e \right] \tag{16}$$

λ for the continuous part of the controller [12] is considered

$$\lambda = \frac{t_0 + \tau}{2t_0 \tau} \tag{17}$$

Thus, the control law can be represented as

$$u = \left(\frac{t_0 \tau}{K_1}\right) \left(\frac{x}{t_0 \tau} + \lambda^2 e\right) + K_d \frac{S}{|S| + \delta} \tag{18}$$

$$\text{With } S = \text{sign}(K_1) [-\dot{x} + 2\lambda e + \lambda^2 \int_0^t e dt] \tag{19}$$

Equations (18) and (19) constitute the controller equations of the SMCr consisting of the tuning parameters and the characteristic parameters of the FOPDT model. The term sign(K<sub>1</sub>) is incorporated in the sliding surface equation (19) which accounts for the process gain and hence the effect of the controller.

**5. SIMULATION**

A mixing tank receiving a hot fluid having mass flow rate F<sub>H</sub>(t) and a cold fluid with mass flow rate F<sub>C</sub>(t) is considered. The temperature sensor is placed 120 ft downstream from the tank where the outflow is measured. The calibration of the temperature sensor is done for a span of 100 to 200 °F. The

reference temperature considered is 150 °F. The stream of heated water is changed from 250 lb/min to 220 lb/min, next to 190 lb/min, then to 160 lb/min, and lastly to 130 lb/min. [1,10,13]

The linearized model of the plant is considered with presumptions as follows:

- Fluid volume in the tank is viewed as consistent.
- Fluid is perfectly mixed in the tank.
- Pipelines and the tank are properly insulated.

The implementation of the designed sliding mode controller is exhibited in this paper.

The steady state values and design parameters for the system are listed in Table 1 [1].

The equations constituting the dynamic model of mixing tank are given as follows [1]:

Energy balance equation of the tank

$$F_H(t)C_{PH}T_H(t) + F_C(t)C_{PC}T_C(t) - (F_H(t) + F_C(t))C_{PT}T_T(t) = V\rho C_{PT} \frac{dT_T(t)}{dt}$$

Tank temperature after transportation delay

$$T_{Td}(t) = T_T(t - t_d)$$

Transportation delay

$$t_d = \frac{LA\rho}{F_H(t) + F_C(t)}$$

Transmitter output

$$\frac{dT(t)}{dt} = \frac{1}{\tau_t} \left[ \frac{T_{Td}(t) - 100}{100} - T(t) \right]$$

Equation for the position of valve

$$\frac{dV_x(t)}{dt} = \frac{1}{\tau_{VP}} [m(t) - V_x(t)]$$

Equation for valve

$$F_C(t) = \frac{500}{60} C_{VF} V_x(t) \sqrt{g\Delta P}$$

Sliding mode controller

$$u = u_c + u_d$$

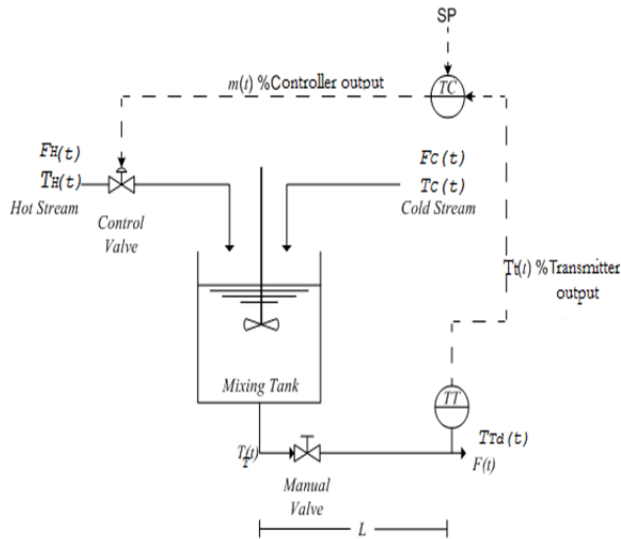


Fig. 2: Control scheme of mixing tank

Table 1: Process parameters and steady-state values [1]

Parameter	Variable	Value
Hot fluid outflow	$F_H$	250.0 lb/min
Cold fluid outflow	$F_C$	190.0 lb/min
Temperature of hot fluid	$T_H$	250.0 °F
Temperature of cold fluid	$T_C$	50.0 °F
Temperature of mixed fluid	$T_T$	150.0 °F
Volume of fluid in tank	$V$	15 ft <sup>3</sup>
Heat capacity at constant pressure of hot fluid	$C_{PH}$	0.8 Btu/lb-°F
Heat capacity at constant pressure of cold fluid	$C_{PC}$	1.0 Btu/lb-°F
Heat capacity at constant volume of mixed fluid	$C_{PT}$	0.9 Btu/lb-°F
Mixed fluid density	$\rho$	62.4 lb/ft <sup>3</sup>
Cross section of pipe	$A$	0.2006 ft <sup>2</sup>
Length of pipe	$L$	120 ft
Coefficient of valve flow	$C_{VF}$	12gpm/psi <sup>1/2</sup>
Reference temperature	$T_{ref}$	150 °F
Time constant of the temperature transmitter	$\tau_T$	0.5 min
Time constant of the control valve	$\tau_{VP}$	0.4 min
Transmitter output	$T_t$	0-1
Specific gravity	$g$	1
Pressure difference across valve	$\Delta P$	16 psi
Controller output	$m(t)$	0-1
Valve position	$V_X$	0.478

The temperature control for mixing tank is studied using two control schemes: i. using conventional PID controller and ii. using Sliding Mode Controller. and the outcome is analyzed. The SMC controller has been designed for a PID sliding surface.

The complete model is developed in MATLAB environment.

To study the robustness property of SMC, disturbance is added to the system as a step variation in flow of hot fluid ( $F_H$ )

as shown in Fig. 3. It is desired to maintain the temperature of the mixed fluid outgoing from the tank ( $T_T$ ) at a temperature of 150 °F and is done by manipulating the flow of the cold fluid ( $F_C$ ).

Fig. 3. shows the temperature response of the mixing tank on application of the Sliding Mode Controller and the PID controller [1] as the hot stream flow ( $F_H$ ) is varied from 250 lb/min to 220 lb/min, next to 190 lb/min, then to 160 lb/min, and lastly to 130 lb/min.

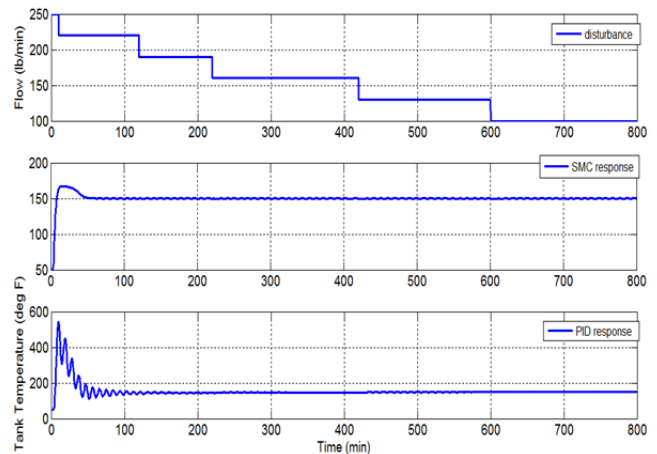


Fig. 3: Disturbance and Response

It is evident from Fig. 3. that the response of the conventional PID controller is not up to the mark owing to the variation in the hot fluid flow. On the other hand the Sliding Mode Controller maintains its robustness and stability despite being affected by the disturbances as mentioned above.

The convergence of error is given in Fig. 4.

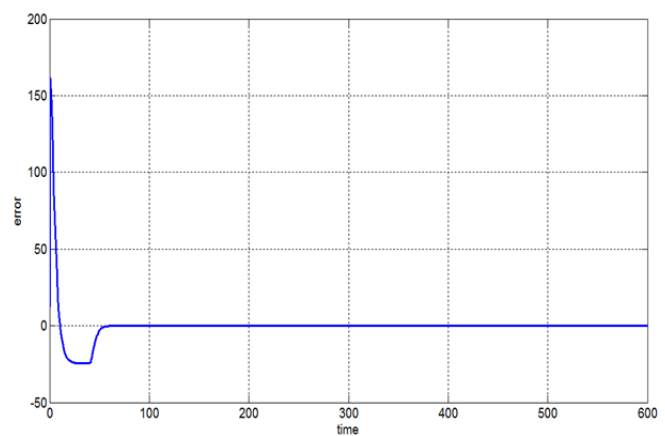


Fig. 4: Error vs Time

## 6. CONCLUSION

The paper demonstrates the design of a controller based on sliding mode technique for an FOPDT model of a thermal mixing tank for temperature control of the tank fluid. The outcome of the work proves that the designed SMC is highly robust as compared to conventional PID controllers for controlling nonlinear chemical processes via reduced order model. The results presented in the paper shows the robustness property of sliding mode technique in presence of nonlinearities and perturbations. The simulation is done for step disturbance added to the flow of hot stream and the controller performance is found to be satisfactory.

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